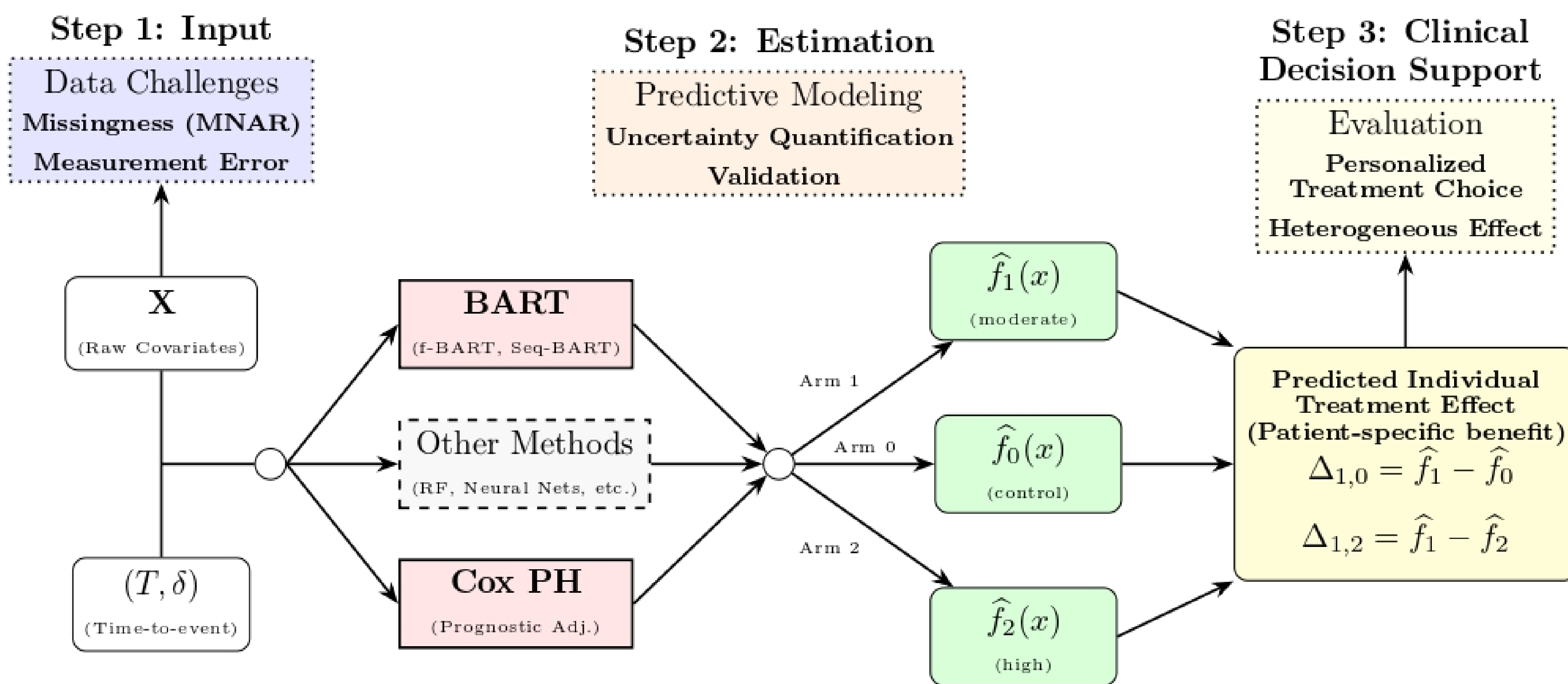


Precision Joint Modeling under Missingness: A Unified Bayesian approach for Predicted Individual Treatment Effects (PITEs)

Pamela M. Chiroque-Solano^{a*}, Thomas Jaki^{a,b}

^a University of Regensburg, Germany ^b University of Cambridge, UK *pamela.chiroque-solano@ur.de

1. Synopsis Chart: Survival Analysis Framework



2. Key Foundations

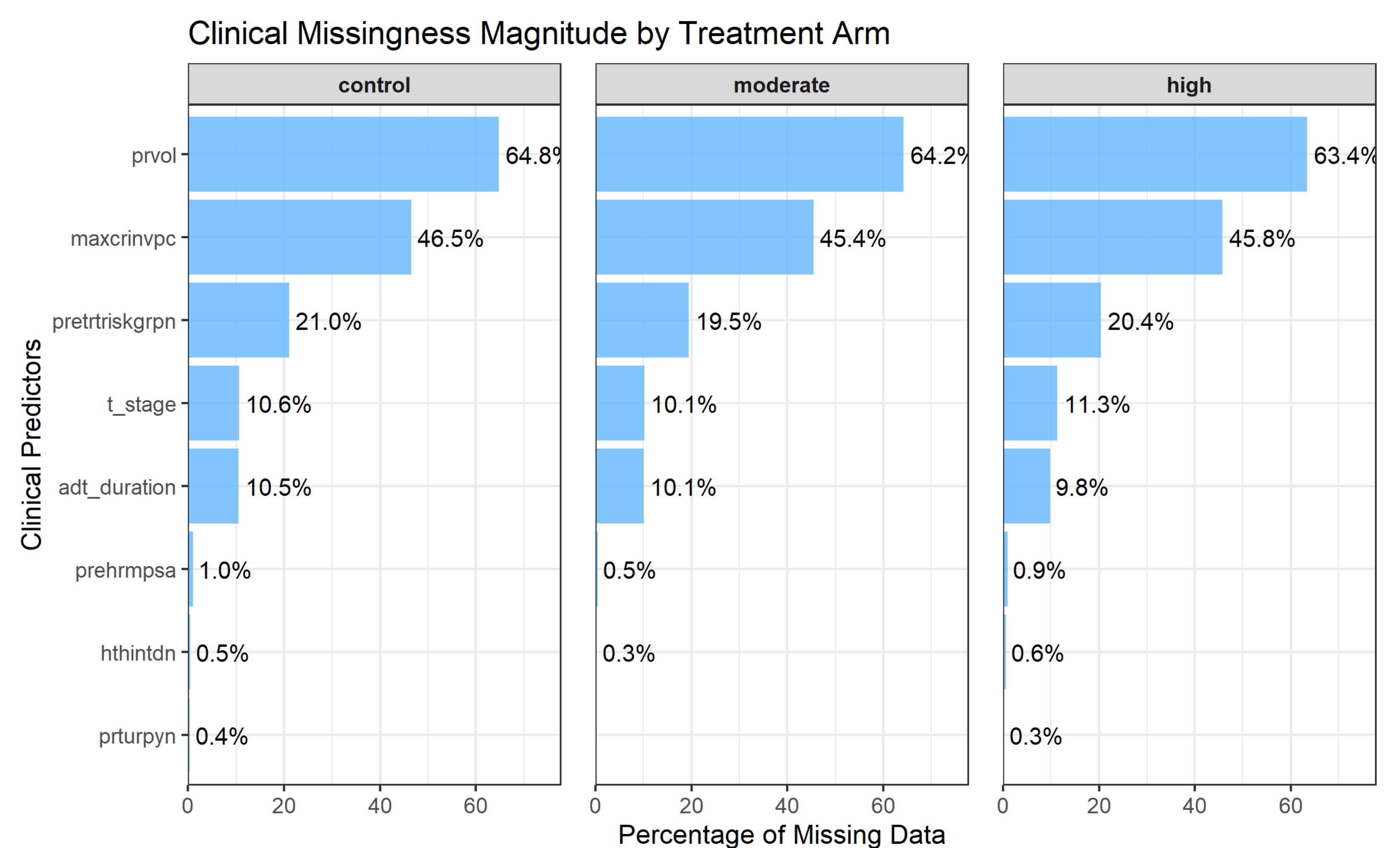
- Notation (subject i)**
True event and Censoring time: T_i^* , C_i
Obs. data: $T_i = \min(T_i^*, C_i)$
 $\delta_i = I(T_i^* \leq C_i)$, right censoring.
Biomarkers $X_i(t)$, Covariates W_i
Goal: infer T_i^* using only (T_i, δ_i)
- Joint Outcome Structure**
 $h_i(t) = h_0(t) \exp\{\text{BART}(W_i) + \alpha b_i\}$
- Informative Missingness (MNAR)**
- From Population to PITEs**
 $\Delta_i(t) = S_i^{(1)}(t) - S_i^{(0)}(t)$
- Bias Propagation in PITE**
Ignoring MNAR \Rightarrow biased $S_i^{(a)}(t)$
 \Rightarrow distorted $\Delta_i(t)$ and treatment decisions

3. A Randomized Clinical Trial

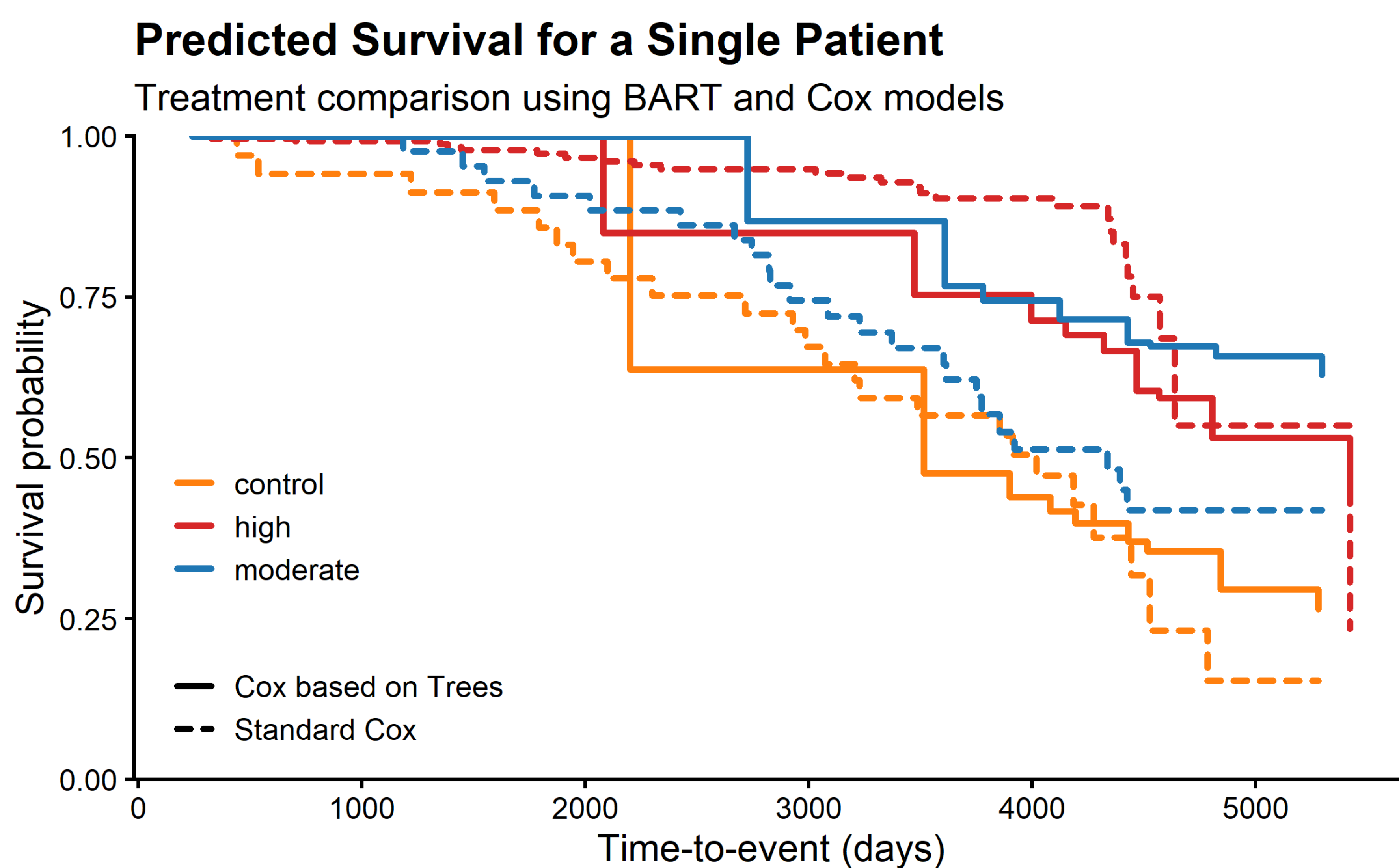
A Hypothetical Clinical Trial Dataset

- Phase III randomized trial**, $n > 3000$, localized prostate cancer
- Radiotherapy arms**: control – moderate – high hypofractionation schedule
- moderate **non-inferior** at > 10 years follow-up.
- Primary Outcome**: Time to biochemical/clinical failure (BCF). **Covariates**:
- Tumor**: $\log(\text{PSA})$, Gleason components, T-stage, prostate volume
- Hormone Treatment**: ADT duration, ADT type
- Clinical**: age, diabetes, hypertension

4. Data Challenges



5. Cox Proportional Hazards Model with Prognostic Adjustment



6. PITEs: $S_i^{(t)}(t) - S_i^{(c)}(t)$

- Missing data**: handled under MCAR via Multiple Imputation \Rightarrow no explicit modeling of missingness mechanism
- Covariates**: included as fixed predictors with no generative model \Rightarrow ignores latent structure
- Survival model**: Cox PH (Cox, 1972), $h_i(t) = h_0(t) \exp(\beta^T W_i)$ with unspecified $h_0(t)$
- Flexible alternative**: BART (50 trees) (Linero et al, 2022; Basak et. al., 2024) \Rightarrow captures nonlinear effects and heterogeneity

7. Work in Progress

- Informative missingness drives bias in PITE/Predictions
- Cox model with unspecified baseline hazard $h_0(t)$
- Longitudinal process $Y_i(t)$ modeled via random effects b_i
- Questions or comments?
Email: pchiroque@gmail.com

8. References

Linero et al. (2022). *Bayesian Survival Tree Ensembles with Submodel Shrinkage*. Bayesian Analysis;
Basak et al. (2024). *Relative Survival Analysis Using Bayesian Decision Tree Ensembles*;
Cox (1972). *Regression Models and Life-Tables*. Journal of the Royal Statistical Society B.